Peer-to-Peer Tangible Goods Rental

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Abstract. We present a game-theoretic model of online tangible private goods rental. Rental mechanisms adhering to this model are classified according to whether their principals and transfer paths are centralized or peer-to-peer. A key issue in all cases is providing incentives for participants to accurately report on quality of the rented good. Our main contribution is a novel mechanism for the situation in which the principal is independent of the owners of goods available in the market and in which goods are transferred directly between renters. We show that it is possible to set mechanism parameters—payments and penalty functions—so that truthful reporting of the quality of goods is a Nash equilibrium.

Keywords: Peer-to-Peer Rental, E-Commerce, Collaborative Consumption, Mechanism Design

1 Introduction

Online tangible private goods rental is an emerging market-based approach to sharing goods among strangers. By providing a medium through which information may be stored and accessed globally, the Internet has greatly extended the feasibility of such markets. Rental enables goods owners to obtain revenue from goods that would otherwise be underutilized, while enabling renters to obtain utility from goods to which they may not otherwise have access.

Commercial services for online tangible goods rental have proliferated in recent years. The claimed benefits of rental are both increased utility for market participants and a reduction of negative environmental externalities [Botsman and Rogers, 2010]. ZipCar (`http://www.zipcar.com`), a renter of automobiles for brief local usage with 270,000 users as of 2009 [Lawson, 2011], boasts a fleet size of 9,000 vehicles, and claims each vehicle obviates the need for 15 personally owned vehicles (`http://www.zipcar.com/is-it/greenbenefits`). Zilok (`http://www.zilok.com`) is a leading rental market of many types of tangible goods, growing at a rate of 25% per year since 2007 [Botsman and Rogers, 2010]. Including offline markets, U.S. consumer rental services increased 27% between 1999 and 2007 to $4.7 billion [Lawson, 2011]. Despite being a growing economic sector, mechanisms for tangible private goods rental have not to our knowledge been previously studied from a game-theoretic perspective.

Market mechanisms vary along two dimensions: the *principal* and the *transfer path* of goods. Examples of markets in industry that vary along these dimensions are provided in §3. The principal sets the mechanism and mediates the market. The transfer path refers to the route that a good takes to move from one renter to the next. We present a framework for tangible private goods rental, within which mechanisms that vary along these dimensions can be modeled and analyzed. Our main contribution is a novel mechanism in which the principal is independent of the owners of goods available in the market and goods are transferred directly between renters.

2 Related Work

As commercial services for online tangible goods rental have emerged only recently, it is not surprising that it has yet to receive much attention in the research literature. Some research has addressed the general social-psychological motivations for non-ownership consumption [Lawson, 2011, Lamberton and Rose, 2012, Belk, 2010, Durgee and O'Connor, 1995], and in particular on social mechanisms for sharing tangible goods [Benkler, 2004]. To our knowledge, tangible goods rental mechanisms have not previously been examined from a game-theoretic perspective.

In foundational work on the relationship between quality and uncertainty in markets, Akerlof [1970] found that trust is a precondition for trading goods with uncertain quality, such as used automobiles. Rental can be reformulated as two transactions in which quality information is asymmetrical, and the difference in purchase price and sale price equals the rental price. For a rental market to succeed, renters must trust that the quality of the good they are to rent is as described.

Sharing and renting are two solution concepts which seek an efficient utilization of goods. Whereas sharing relies upon social mechanisms, renting relies upon market mechanisms. The focus of some sharing research considers the pooling and shared consumption of private goods [Lamberton and Rose, 2012, Belk, 2010], a mechanism which treats private goods as common within a group of agents. Such sharing arrangements may be susceptible to the Tragedy of the Commons [Hardin, 1968], in which each agent's rational best response is to use, and potentially deplete, the entire pool themselves. The model in this paper differs in that the good is modeled as a perfectly excludable private good, used by a single agent at any one time.

Belk [2010] describes the sharing of joint possessions as carrying socially enforced responsibilities to not cause damage, to not overuse, and to clean up after the use of goods. The design goal of rental mechanisms is a market-based enforcement of the same responsibilities.

Benkler [2004] conjectures that some reasons social-based sharing mechanisms are more prevalent than market-based sharing mechanisms are lower transaction costs, better information, and stronger motivations for owners. We similarly observe from the model in this paper the need to minimize transaction costs and elicit truthful reports from market participants in order to maximize social welfare.

3 Rental Mechanism Classes

We classify mechanisms along two dimensions: whether the principal is also the owner of goods available in the market (*central* if so, *peer-to-peer* otherwise), and whether goods are transferred to the owner between renters (*central* if so, *peer-to-peer* otherwise). Figure 1 presents a brief summary of the four online tangible goods rental mechanism classes.

Rental / *Transfer*	**Central**	**Peer-to-Peer**
Central	Goods owner mediates the market. Goods are transferred between renters by way of the owner.	Market mediator is a third party to the transaction. Goods are transferred between renters by way of the owner.
Peer-to-Peer	Goods owner mediates the market. Goods are transferred directly between renters.	Market mediator is a third party to the transaction. Goods are transferred directly between renters.

Fig. 1. Brief description of each of the four mechanism classes.

The first mechanism class is *central rental, central transfer*, in which renters interact directly with the owner during every stage of the transaction. When a good is returned to the owner, it is the owner who reports the current quality of the good. We show that the best response of the owner is to report quality untruthfully. An example of such a market is Chegg (`http://www.chegg.com`), which primarily rents textbooks.

The second mechanism class is *central rental, peer-to-peer transfer*, in which goods are not typically handled by the owner between rentals, but the owner sets the mechanism and mediates the market. This market class differs from the previous class in that reporting damage done to a good is the responsibility of the renter. An example of such a market is ZipCar, which rents vehicles strategically located near clusters of renters. A renter reserves a vehicle through ZipCar's web site, then picks it up from the unmonitored parking lot in which it is located. When finished using the vehicle, the renter returns it to the same parking lot. It is the responsibility of the renter to inspect the vehicle thoroughly before using it. If the renter fails to report damage that occurred previous to her use of the vehicle, she becomes liable for that damage. We show that, as market mediator, the owner has the opportunity to falsify reports, and thus the incentive properties of such a mechanism are equivalent to the central rental, central transfer mechanism.

The next mechanism class is *peer-to-peer rental, central transfer*. In such a mechanism, renters interact with a third-party market mediator to find and reserve goods before interacting with the owner to obtain the goods. A benefit of this market class over central rental mechanisms is that renters here may have intramarket recourse against untruthful reports by owners through the use of a reputation mechanism, which may incentivize the owner to report truthfully in equilibrium. An example of such a market is Zilok, a market for rental of all types of tangible goods.

The final mechanism class is *peer-to-peer rental, peer-to-peer transfer*, in which renters interact with a third-party market mediator to find and reserve a good before interacting with the previous renter to obtain the good, removing the owner from the process with the exception of introducing the good. We were unable to find any commercial or prior literature examples of such a market, thus this appears to be a novel formulation.

4 Model

We present a general model of tangible private goods rental. The various mechanism classes are distinct designs which conform to this model.

Utility derived from tangible goods is a function of several attributes, one of which is time. Some goods, such as books, have high utility until the good is consumed, at which point utility drops. Other goods, such as lawn mowers, have a utility that cycles between high and low over time. Another attribute of tangible goods that affects utility is quality. As a good is used, its quality is degraded with some probability per unit of time. Our model adopts the assumption that quality is monotonically decreasing. Another simplifying assumption we make is that agents are able to perfectly assess a good's quality on a commonly understood discrete-valued scale.

All market participants are rational, self-interested, and risk-neutral. The *principal*, m, is the designer and mediator of the mechanism. The owner o of good g may or may not also be m. Utility for g is one-time or cyclic. There is a stream of renters $\rho_0, \ldots, \rho_i, \ldots, \rho_n = P$ who sequentially use g.

Renter ρ_i uses g during time period i. At any point in time, g has integer quality $q \in \{0, \ldots, q^{\max}\} = Q$ such that $q^{\max} \geq 1$. At the beginning of time period i, g has quality q_i^{in}. At the end of time period i, g has quality $q_i^{out} \leq q_i^{in}$. The probability of degrading from q^{in} to q^{out} in one time period is $\delta_{q^{in} \to q^{out}} \in [0, 1]$. For all q^{in}, the constraint $\sum_{q^{out} \leq q^{in}} \delta_{q^{in} \to q^{out}} = 1$ ensures a well-defined probability distribution. We denote by $\hat{q}$ a quality report submitted to m. The symbol $\hat{q}^{\ddagger}$ designates a truthful report, where $\hat{q} = q$. The choice of reported quality that results in the highest expected utility for the reporter is denoted $\hat{q}^*$.

To simplify notation and reasoning, we assume that the renters value using g equally. The value v_q of using a good with quality $q^{in} = q$ at the beginning of the time period decreases monotonically with q. For calibration, we set $v_0 = 0$.

The cost of physically transferring g from one agent to another is θ_t, and is borne by the receiving agent. No degradation occurs during transfer. The cost of processing the transfer of g is θ_p, which is meant to reflect in part the cost of effort to assess the quality of the good. θ_p is less than θ_t because processing is a step in transferring the good.

A market does not exist in isolation. Disregarding property rights, g can be sold by the holding agent for an amount that decreases monotonically with g's quality, c_q, with $c_0 = 0$. At the beginning of the game, o has purchased g at cost $c_{q^{\max}}$.

Consider first a basic mechanism in which ρ pays some amount to o in exchange for renting g. The payment $p_{\hat{q}^{in}}$ decreases monotonically with reported incoming quality, $\hat{q}^{in}$, with $p_0 = 0$. A goal of the mechanism is to incentivize ρ to return g when finished using it. When ρ holds g, she has the opportunity to sell it for the amount c_q. When ρ returns g, she receives 0 additional utility, whereas she receives c_q additional utility when she sells g. Because $c_q > 0$ for $q > 0$ by definition, it is impossible to incentivize ρ to return g with this basic payment mechanism. This is in line with the intuitive notion that the same mechanism for selling goods cannot also be used to rent goods.

A solution to the return problem is to include a deposit, by which ρ loses d when g is not returned at the end of the time period. A deposit incentivizes ρ to return g when $c < d$. This is in line with the mechanisms of observed markets. Zilok requires renters post a deposit in order to rent a good, and Chegg holds monetary account information in order to charge the difference between the purchase price and the rental price in the event a book is not returned (`http://www.chegg.com/generalpolicies/`).

Let u_o denote the utility of the owner, and u_ρ the utility of a renter. The functions u_o and u_ρ may differ between mechanisms. We define social welfare w to be the sum of the expected utilities of all market participants,

$$w = \mathbb{E}[u_o] + \sum_{\rho \in P} \mathbb{E}[u_\rho].$$

With individual rationality constraints ensuring that expected utility is non-negative, social welfare is proportional to the number of renters. Thus, increasing the number of renters increases social welfare. What prevents the number of renters from reaching infinity is the probabilistic degradation of the quality of g. When the probability of degradation is higher, the expected lifespan of the good is lower, and thus the number of renters expected to use the good is lower.

Consider the addition of a reward to the mechanism, by which ρ receives an amount of utility that decreases monotonically with the value of reported outgoing quality, $r_{\hat{q}^{out}}$, with $r_0 = 0$. Assuming ρ has the ability to control her probability of degrading q at some cost of effort, she will respond to the reward by increasing the care with which g is handled by an amount that is a function of effort, thus decreasing the probability that q degrades.

Social welfare can be further increased by minimizing the cost of transferring a good between renters. This provides motivation to the peer-to-peer transfer of goods, in which goods are transferred directly from one renter to the next instead of being routed through the owner, reducing the number of transfers by approximately half.

5 Central Rental Mechanisms

In central rental mechanisms, m and o are the same agent. Treating rental as a one-shot interaction, this provides o with substantial power to manipulate the market to her benefit. Owner o reports $\hat{q}$ in the central transfer mechanism, while ρ reports $\hat{q}$ in the peer-to-peer transfer mechanism. Because o is also m, o has the opportunity to falsify reports by ρ, and thus the two mechanisms may both be modeled as if o reports $\hat{q}$. This scenario is represented as a two-player sequential game in which o chooses $\hat{q}_i^{in}$, ρ_i obtains, uses, and chooses whether to return g, then o chooses $\hat{q}_i^{out}$.

An available strategy of o is to white-wash the quality of g. White-washing is a strategy commonly considered for cleansing agent reputations [Feldman et al., 2006]. This strategy is applied here to a good, which enables o to make independent reports of quality between renters of the same good. Applying this strategy, o's expected utility when reporting $\hat{q}_i^{out} = 0$ is greater than o's expected utility when reporting $\hat{q}_i^{out} > 0$ by $r_{\hat{q}_i^{out}}$, which is non-negative by definition, so $\hat{q}_i^{out} = 0$ and white-washing g is the best response by o when ρ_i returns g.

It is in the best interest of o to incentivize ρ to return g, which is accomplished with a deposit d. Even though ρ does not expect to receive reward, the alternative choice of losing $c_q - d$ incentivizes ρ to return g. Whereas the central rental mechanisms may be functional in that the renter will return the good, ρ is effectively charged based on completely degrading the good, and so rental will not occur unless the value is exceedingly high. Moreover, even when rental does take place, ρ lacks any incentive to take care in handling g to preserve its quality.

A more extreme result occurs when the owner reneges on the promised return of the deposit. In this case, the best response of the renter is to sell the good instead of returning it, and the market breaks down in equilibrium.

We emphasize that these conclusions rely on the fact that in a one-shot interaction, the owner can act with impunity in white-washing reputations and even reneging on returning deposits. In reality, interactions may repeat or play out publicly over time, in which case other forces (e.g., through external reputation channels), not modeled here, may substantially check the owner's power. With such extensions, central rental may work well in practice. Thus, we consider more salient our positive results on peer-to-peer markets, where we can achieve desirable properties even in this simple one-shot interaction model.

6 Peer-to-Peer Rental Mechanisms

Recall that peer-to-peer rental mechanisms differ from central rental mechanisms in that the market mediator does not also own the goods available in the market. The arguments of the previous section suggest that central rental mechanisms may fail to operate well because the renter lacks any control over quality evaluations. Whereas this may be alleviated in the central case through adoption of a reputation mechanism [Dellarocas, 2003, Friedman et al., 2007, Josang et al., 2007], our focus here is to examine whether central aggregation of reputation information can be avoided in a peer-to-peer context. Our hypothesis is that peer-to-peer rental, central transfer mechanisms may incentivize o to report truthfully by offering intramarket recourse to renters. We thus focus on the design of a peer-to-peer rental, peer-to-peer transfer mechanism which does not appeal to explicit reputation-based methods.

In the proposed peer-to-peer rental, peer-to-peer transfer mechanism, ρ_0 obtains g from o, and ρ_{i+1} obtains g from ρ_i. When ρ_i receives g, she reports $\hat{q}_i^{in}$ before using g. When finished, she reports $\hat{q}_i^{out}$ before transferring g to ρ_{i+1}. Figure 2 depicts this sequence of events.

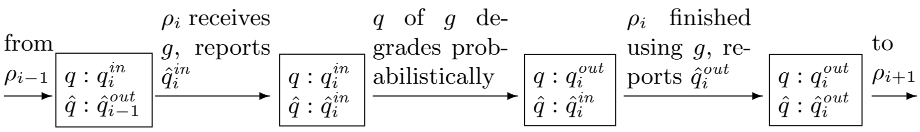

Fig. 2. Sequence of events in the peer-to-peer rental, peer-to-peer transfer mechanism from the perspective of ρ_i. Each box represents the state of g at each stage, where q is true quality, and $\hat{q}$ is reported quality.

As in other mechanisms, ρ_i pays $p_{\hat{q}_i^{in}}$ and receives value $v_{q_i^{in}}$. Two *deviation penalties* penalize ρ_i for reporting $\hat{q}$ differently from ρ_{i-1} or ρ_{i+1}. First is the penalty $n^{in}_{\Delta\hat{q}}$ for reducing $\hat{q}$ by $\Delta\hat{q}$ upon reporting $\hat{q}^{in}$, and second is the penalty $n^{out}_{\Delta\hat{q}}$ to ρ_i when ρ_{i+1} reduces $\hat{q}$ by $\Delta\hat{q}$. These penalties are strictly positive when $\Delta\hat{q} > 0$, and are zero when $\Delta\hat{q} = 0$. In addition, $n^{in}_{\Delta\hat{q}} = 0$ when $\hat{q}^{in} = 0$ regardless of the value of $\Delta\hat{q}$. Also as in other mechanisms, a *deposit* $d_{\hat{q}_i^{in}}$ is taken from ρ_i for not returning g. A reward $r_{\hat{q}_{i+1}^{in}}$ is given to ρ_i, for which $r_{\hat{q}} < p_{\hat{q}}$. Note that when $\hat{q}_i^{in} = 0$ or $\hat{q}_i^{out} = 0$, g is immediately returned to o.

In the following, we analyze the decision faced by ρ_i taking as fixed the strategies of other agents; that is, we examine ρ_i's *best response*. The outcome to ρ_i is conditionally independent of other agents, given its own action as well as those immediately prior $(i-1)$ and succeeding $(i+1)$. The reporting actions of agent $i-1$ are directly observed, so we can formulate ρ_i's decision problem in terms of a best response to ρ_{i+1}'s strategy in reporting $\hat{q}_{i+1}^{in}$. The mixed strategy of ρ_{i+1} takes the form:

$$\Pr(\hat{q}_{i+1}^{in} \mid q_i^{out}, \hat{q}_i^{out}).$$

Hereafter, we take the given variables q_i^{out} and $\hat{q}_i^{out}$ as implicit in describing ρ_{i+1}'s strategy. Given the ρ_{i+1} strategy $\Pr(\hat{q}_{i+1}^{in})$, the expected utility of ρ_i upon reporting $\hat{q}_i^{out}$ is

$$\begin{aligned}\mathbb{E}[u_{\rho_i} \mid \hat{q}_{i-1}^{out}, q_i^{in}, \hat{q}_i^{in}, q_i^{out}, \hat{q}_i^{out}] = {} & v_{q_i^{in}} - p_{\hat{q}_i^{in}} - n^{in}_{\hat{q}_{i-1}^{out}-\hat{q}_i^{in}} - \theta_t - \theta_p \\ & + \sum_{\hat{q}_{i+1}^{in} \le \hat{q}_i^{out}} \Pr(\hat{q}_{i+1}^{in})[r_{\hat{q}_{i+1}^{in}} - n^{out}_{\hat{q}_i^{out}-\hat{q}_{i+1}^{in}}].\end{aligned} \quad (1)$$

We also take as implicit the variables $\hat{q}_{i-1}^{out}$, q_i^{in}, $\hat{q}_i^{in}$, and q_i^{out} in all following uses. Expected utility of the renter upon reporting incoming quality is given without reference to d. The expected utility of ρ_i upon reporting $\hat{q}_i^{in}$ is

$$\begin{aligned}\mathbb{E}[u_{\rho_i} \mid \hat{q}_{i-1}^{out}, q_i^{in}, \hat{q}_i^{in}] = v_{q_i^{in}} - p_{\hat{q}_i^{in}} - n^{in}_{\hat{q}_{i-1}^{out} - \hat{q}_i^{in}} - \theta_t - \theta_p \\ + \sum_{q_i^{out} \leq q_i^{in}} \sum_{\hat{q}_{i+1}^{in} \leq \hat{q}_i^{out*}} \delta_{q_i^{in} \to q_i^{out}} \Pr(\hat{q}_{i+1}^{in})[r_{\hat{q}_{i+1}^{in}} - n^{out}_{\hat{q}_i^{out*} - \hat{q}_{i+1}^{in}}],\end{aligned}$$

with given variables $\hat{q}_{i-1}^{out}$ and q_i^{in} taken as implicit in following uses. If reports are truthful, the expected utility of o is

$$\begin{aligned}\mathbb{E}[u_o \mid q_i^{in}] &= p_{q_i^{in}} + \sum_{q_i^{out} \leq q_i^{in}} \delta_{q_i^{in} \to q_i^{out}} (\mathbb{E}[u_o \mid q_{i+1}^{in} = q_i^{out}] - r_{q_{i+1}^{in}}), \\ \mathbb{E}[u_o \mid q^{\max}] &= -c_{q^{\max}} - \theta_t - \theta_p + \mathbb{E}[u_o \mid q_0^{in} = q^{\max}].\end{aligned}$$

The goal of incentivizing ρ_i to report truthfully is expressed by

$$\begin{aligned}\hat{q}_i^{out*} = \hat{q}_i^{out\ddagger} = \min\{q_i^{out}, \hat{q}_i^{in}\}, \\ \hat{q}_i^{in*} = \hat{q}_i^{in\ddagger} = \min\{q_i^{in}, \hat{q}_{i-1}^{out}\}.\end{aligned}$$

Given the exogenous parameters v, δ, θ_t, and θ_p, we set the mechanism parameters p, n^{in}, n^{out}, d, and r in order to incentivize truthful reports, the return of the good, and voluntary participation. Truthful reports are strictly incentivized by the constraints

$$\mathbb{E}[u_{\rho_i} \mid \hat{q}_i^{out} = \hat{q}_i^{out\ddagger}] > \mathbb{E}[u_{\rho_i} \mid \hat{q}_i^{out} = \neg\hat{q}_i^{out\ddagger}] \; ; \; \forall_{\hat{q}_i^{in} > 0} \forall_{q_i^{out}} \forall_{\neg\hat{q}_i^{out\ddagger} \leq \hat{q}_i^{in}}, \quad (2)$$

$$\mathbb{E}[u_{\rho_i} \mid \hat{q}_i^{in} = \hat{q}_i^{in\ddagger}] > \mathbb{E}[u_{\rho_i} \mid \hat{q}_i^{in} = \neg\hat{q}_i^{in\ddagger}] \; ; \; \forall_{\hat{q}_{i-1}^{out} > 0} \forall_{q_i^{in}} \forall_{\neg\hat{q}_i^{in\ddagger} \leq \hat{q}_{i-1}^{out}}, \quad (3)$$

where $\neg\hat{q}_i^{out\ddagger}$ is an outgoing quality report that is not truthful, and $\neg\hat{q}_i^{in\ddagger}$ is an incoming quality report that is not truthful. In the case of truthful reports, ρ_i is incentivized to return g by the constraints

$$r_{\hat{q}_{i+1}^{in\ddagger}} > c_{q_i^{out}} - d_{\hat{q}_i^{in}} \; ; \; \forall_{\hat{q}_i^{in} > 0} \forall_{\hat{q}_{i+1}^{in} \leq \hat{q}_i^{in}}. \quad (4)$$

Individual rationality constraints for ρ_i, again for the case of truthful reports, are

$$\mathbb{E}[u_{\rho_i} \mid \hat{q}_i^{in} = \hat{q}_i^{in\ddagger}, \hat{q}_i^{out} = \hat{q}_i^{out\ddagger}] \geq 0 \; ; \; \forall_{q_i^{in}}, \quad (5)$$

and individual rationality constraints for o are

$$\mathbb{E}[u_o \mid q^{\max}] \geq 0 \; ; \; \forall_{q^{\max}}, \quad (6)$$

$$\mathbb{E}[u_o \mid q_i^{in}] \geq 0 \; ; \; \forall_{q_i^{in}}. \quad (7)$$

The inequalities (6) incentivize initial participation, and (7) incentivizes continued participation upon every subsequent rental.

Proofs of the following propositions are available in the Appendix. Proposition 1 establishes that ρ_i's best response to a given value for the incoming report of ρ_{i+1} is to match that report. In particular, given that the next renter reports truthfully, the current renter should report outgoing quality truthfully.

Proposition 1 *Given $\hat{q}^{in}_{i+1}$, the optimal outgoing report of ρ_i is $\hat{q}^{in}_{i+1}$, that is, $\hat{q}^{out*}_i = \hat{q}^{in}_{i+1}$.*

This result further entails that the constraints (2) on truthful outgoing reports are automatically satisfied when the constraints (3), which incentivize truthful incoming reports, are satisfied.

Proposition 2 states that, given that the outgoing report of the previous renter is truthful, the incoming report of the next renter is truthful, and the incoming quality of g is non-zero, then a specified set of constraints on mechanism parameters is sufficient for truthful reporting of incoming quality. These constraints incentivize the renter to report truthfully by ensuring that the reduced payment of a lower report is offset by a penalty for reducing reported quality and the reduction of expected future reward.

Proposition 2 *Suppose $\hat{q}^{out}_{i-1} = \hat{q}^{out\ddagger}_{i-1}$, $\hat{q}^{in}_{i+1} = \hat{q}^{in\ddagger}_{i+1}$, and $q^{in}_i > 0$, and that for all nontruthful incoming reports $0 < \neg\hat{q}^{in\ddagger}_i < \hat{q}^{in\ddagger}_i$, the penalties and payments satisfy the following constraints:*

$$n^{in}_{\hat{q}^{out\ddagger}_{i-1} - \neg\hat{q}^{in\ddagger}_i} + \sum_{q^{out}_i \le q^{in}_i} \delta_{q^{in}_i \to q^{out}_i} [r_{\hat{q}^{in\ddagger}_{i+1}} - r_{\min\{\neg\hat{q}^{in\ddagger}_i, q^{out}_i\}}] > p_{\hat{q}^{in\ddagger}_i} - p_{\neg\hat{q}^{in\ddagger}_i}, \quad (8)$$

$$v_{q^{in}_i} - p_{\hat{q}^{in\ddagger}_i} + \sum_{q^{out}_i \le q^{in}_i} \delta_{q^{in}_i \to q^{out}_i} r_{\hat{q}^{in\ddagger}_{i+1}} > 0. \quad (9)$$

Then the optimal incoming report is truthful, $\hat{q}^{in*}_i = \hat{q}^{in\ddagger}_i$.

Proposition 3 states that renters will report truthfully when the incoming quality of g is zero, regardless of the reports of the previous and next renters.

Proposition 3 *Given $q^{in}_i = 0$, the optimal incoming report of ρ_i is also zero, that is, $\hat{q}^{in*}_i = \hat{q}^{in\ddagger}_i$.*

Taken together, the three Propositions establish that reporting truthfully is a Nash equilibrium, assuming the inequalities of Proposition 2 are satisfied.

We illustrate the calculation of mechanism parameters for an example scenario. Consider renting the book *Artificial Intelligence: A Modern Approach, 3d Edition* (*AIMA*), by Stuart Russell and Peter Norvig, for 120 days. On Amazon, the sale prices of *AIMA* are \$117 in new/like-new condition and \$85 in good/acceptable condition. In our model, new/like-new condition corresponds to $q = 2$, and good/acceptable condition corresponds to $q = 1$. According to prices aggregated from six textbook rental web sites, a new or like-new conditioned *AIMA* costs \$0.48 per day to rent. According to listed prices on a used book site, the value of this book diminishes by 7% between new/like-new/very-good and good/acceptable conditions. We assume that the consumer's value of using the book is somewhat higher than the observed price of rental. The book is 10.4 by 8.4 by 1.9 inches in size, and weighs 4.4 pounds, which costs $11 to ship domestically via U.S. Postal Service. We further set $q^{\max} = 2$ to maintain a presentable number parameters. Table 1 presents the exogenous parameters of this scenario.

$\mathbf{c_2}$	$\mathbf{c_1}$	$\mathbf{v_2}$	$\mathbf{v_1}$	$\delta_{\mathbf{2\to2}}$	$\delta_{\mathbf{2\to1}}$	$\delta_{\mathbf{2\to0}}$	$\delta_{\mathbf{1\to1}}$	$\delta_{\mathbf{1\to0}}$	$\theta_{\mathbf{t}}$	$\theta_{\mathbf{p}}$
117	85	58	54	0.9	0.08	0.02	0.9	0.1	11	1

Table 1. Exogenous parameters for the *AIMA* rental example.

We calculate mechanism parameters through linear programming, to maximize participant expected utility subject to mechanism and incentive constraints. Specifically, we incorporate the constraints (3), (4), (5), (6), (7), and the base mechanism constraints. We present result for two objective functions, owner expected utility when $q = q^{\max}$ and renter expected utility when $q^{in} = q^{\max}$, yielding two sets of candidate mechanism parameters. The mechanism parameters presented in Table 2 satisfy the specified constraints, assuming that the next renter reports truthfully, $\hat{q}^{in}_{i+1} = \hat{q}^{in\ddagger}_{i+1}$. In other words, we exhibit mechanism parameters under which rational agents have the incentives to participate, and report truthful qualities in Nash equilibrium.

Objective	$\mathbf{p_2}$	$\mathbf{p_1}$	$\mathbf{r_2}$	$\mathbf{r_1}$	$\mathbf{d_2}$	$\mathbf{d_1}$	$\mathbf{n_2^{in}}$	$\mathbf{n_1^{in}}$	$\mathbf{n_2^{out}}$	$\mathbf{n_1^{out}}$
$\max \mathbb{E}[\mathbf{u_o}]$	70.79	41.99	32.01	0.01	84.99	116.99	0.02	0.01	0.02	0.01
$\max \mathbb{E}[\mathbf{u}_\rho]$	14.35	14.32	14.33	0.01	102.67	116.99	0.02	0.01	0.09	0.01

Table 2. Mechanism parameters incentivizing the truthful reporting equilibrium, return of g, and voluntary participation.

Next, we examine the robustness of the mechanism parameter settings by supposing that the next renter reports incoming quality $\hat{q}^{in}_{i+1}$ truthfully with only some probability, m. The full reporting strategy is described conditional on all combinations of true quality and outgoing ρ_i reports in Table 3.

We calculated mechanism parameters for $m \in \{1, 0.99, 0.9, 0.8, \ldots, 0.1, 0.01\}$. As m was decreased, the linear constraints used in each optimization were accumulated for use in subsequent optimizations. At each level of m, we verified that the resulting parameters were valid for all greater values of m. Calculated values when $m = 0.5$ and $m = 0.01$ are presented in Table 4.

$\mathbf{q_i^{out}}$	$\mathbf{\hat{q}_i^{out}}$	$\mathbf{\hat{q}_{i+1}^{in}}$	$\Pr(\mathbf{\hat{q}_{i+1}^{in}} \mid \mathbf{q_i^{out}}, \mathbf{\hat{q}_i^{out}})$
0	0	0	1
0	1	0	m
0	1	1	$1-m$
0	2	0	m
0	2	1	$0.75(1-m)$
0	2	2	$0.25(1-m)$
1	0	0	1
1	1	0	$1-m$
1	1	1	m
1	2	0	$0.5(1-m)$
1	2	1	m
1	2	2	$0.5(1-m)$
2	0	0	1
2	1	0	$1-m$
2	1	1	m
2	2	0	$0.25(1-m)$
2	2	1	$0.75(1-m)$
2	2	2	m

Table 3. Probabilities of $\hat{q}_{i+1}^{in}$ given q_i^{out} and $\hat{q}_i^{out}$, where m controls truthfulness.

7 Conclusions

Our game-theoretic examination of tangible private goods rental suggests that incentives for accurate quality reporting depend on the organization of the market as well as key mechanism parameters. For mechanisms in which the owner of goods is also the principal of the market, we observed that the owner is motivated to misrepresent the quality of the good after the renter has returned it. More constructive is our finding that mechanisms in which the principal is a third-party to the transaction can induce incentive compatibility. The version of such a mechanism in which the good is transferred indirectly between renters may achieve incentive compatibility with the implementation of reputation-based mechanisms, which give intramarket recourse to renters.

The main contribution of this work is a novel mechanism in which the principal is a third-party and goods are transferred directly between renters. We showed that with this mechanism, incentive compatibility can be achieved without explicit management of participant reputation. This mechanism minimizes the number of times the good is transferred, typically a significant cost for online tangible goods rental mechanisms. A drawback of the approach is that the mechanism parameters (payments and penalties) must be tailored to the environment, which may be informationally expensive due to the need to assess exogenous domain-specific features. Future work should evaluate the sensitivity of mechanism properties to such factors, as well as the potential and implications of multiple equilibria.

m	Objective	$\mathbf{p_2}$	$\mathbf{p_1}$	$\mathbf{r_2}$	$\mathbf{r_1}$	$\mathbf{d_2}$	$\mathbf{d_1}$	$\mathbf{n_2^{in}}$	$\mathbf{n_1^{in}}$	$\mathbf{n_2^{out}}$	$\mathbf{n_1^{out}}$
0.5	$\max \mathbb{E}[\mathbf{u_o}]$	42.00	41.98	0.05	0.01	116.98	117.00	0.02	0.01	0.02	0.01
	$\max \mathbb{E}[\mathbf{u}_\rho]$	9.72	9.71	5.08	0.01	114.64	117.00	0.02	0.01	1.45	0.01
0.01	$\max \mathbb{E}[\mathbf{u_o}]$	41.99	41.97	0.05	0.03	116.99	117.01	0.02	0.01	0.02	0.01
	$\max \mathbb{E}[\mathbf{u}_\rho]$	9.72	9.71	5.06	0.01	118.77	118.81	0.02	0.01	1.91	1.81

Table 4. Calculated mechanism parameter values optimized for maximum owner expected utility when $q = q^{\max}$ and renter expected utility when $q^{in} = q^{\max}$ as m is varied.

Appendix A: Proof of Proposition 1

Given $\hat{q}_{i+1}^{in}$, expected utilities of ρ_i upon reporting $\hat{q}_i^{out}$ are

$$\begin{aligned}
\mathbb{E}[u_{\rho_i} \mid \hat{q}_{i+1}^{in}, \hat{q}_i^{out} = \hat{q}_{i+1}^{in}] &= v_{q_i^{in}} - p_{\hat{q}_i^{in}} - n^{in}_{\hat{q}_{i-1}^{out} - \hat{q}_i^{in}} - \theta_t - \theta_p \\
&\quad + r_{\hat{q}_{i+1}^{in}} \\
\mathbb{E}[u_{\rho_i} \mid \hat{q}_{i+1}^{in}, \hat{q}_i^{out} > \hat{q}_{i+1}^{in}] &= v_{q_i^{in}} - p_{\hat{q}_i^{in}} - n^{in}_{\hat{q}_{i-1}^{out} - \hat{q}_i^{in}} - \theta_t - \theta_p \\
&\quad + r_{\hat{q}_{i+1}^{in}} - n^{out}_{>\hat{q}_{i+1}^{in} - \hat{q}_{i+1}^{in}} \\
\mathbb{E}[u_{\rho_i} \mid \hat{q}_{i+1}^{in}, \hat{q}_i^{out} < \hat{q}_{i+1}^{in}] &= v_{q_i^{in}} - p_{\hat{q}_i^{in}} - n^{in}_{\hat{q}_{i-1}^{out} - \hat{q}_i^{in}} - \theta_t - \theta_p \\
&\quad + r_{<\hat{q}_{i+1}^{in}}.
\end{aligned}$$

$\hat{q}_i^{out*} > \hat{q}_{i+1}^{in}$ when

$$\begin{aligned}
\mathbb{E}[u_{\rho_i} \mid \hat{q}_{i+1}^{in}, \hat{q}_i^{out} > \hat{q}_{i+1}^{in}] &> \mathbb{E}[u_{\rho_i} \mid \hat{q}_{i+1}^{in}, \hat{q}_i^{out} = \hat{q}_{i+1}^{in}] \\
0 &> n^{out}_{>\hat{q}_{i+1}^{in} - \hat{q}_{i+1}^{in}}.
\end{aligned}$$

$n^{out} \geq 0$ by definition, and thus $\hat{q}_i^{out*} \leq \hat{q}_{i+1}^{in}$. $\hat{q}_i^{out*} < \hat{q}_{i+1}^{in}$ when

$$\begin{aligned}
\mathbb{E}[u_{\rho_i} \mid \hat{q}_{i+1}^{in}, \hat{q}_i^{out} < \hat{q}_{i+1}^{in}] &> \mathbb{E}[u_{\rho_i} \mid \hat{q}_{i+1}^{in}, \hat{q}_i^{out} = \hat{q}_{i+1}^{in}] \\
r_{<\hat{q}_{i+1}^{in}} &> r_{\hat{q}_{i+1}^{in}}.
\end{aligned}$$

$r_{<\hat{q}_{i+1}^{in}} < r_{\hat{q}_{i+1}^{in}}$ by definition, and thus $\hat{q}_i^{out*} = \hat{q}_{i+1}^{in}$.

Appendix B: Proof of Proposition 2

Given $\hat{q}_{i-1}^{out} = \hat{q}_{i-1}^{out\ddagger}$, $\hat{q}_{i+1}^{in} = \hat{q}_{i+1}^{in\ddagger}$, and $q_i^{in} > 0$, the expected utilities of ρ_i given $\hat{q}_i^{in}$ are

$$
\begin{aligned}
\mathbb{E}[u_{\rho_i} \mid \hat{q}_i^{in} = \hat{q}_i^{in\ddagger}] &= v_{q_i^{in}} - p_{\hat{q}_i^{in\ddagger}} - n^{in}_{\hat{q}_{i-1}^{out\ddagger} - \hat{q}_i^{in\ddagger}} - \theta_t - \theta_p \\
&\quad + \sum_{q_i^{out} \leq q_i^{in}} \delta_{q_i^{in} \to q_i^{out}} r_{\hat{q}_{i+1}^{in\ddagger}} \\
\mathbb{E}[u_{\rho_i} \mid \neg\hat{q}_i^{in\ddagger} \in (0, \hat{q}_i^{in\ddagger})] &= v_{q_i^{in}} - p_{\neg\hat{q}_i^{in\ddagger}} - n^{in}_{\hat{q}_{i-1}^{out\ddagger} - \neg\hat{q}_i^{in\ddagger}} - \theta_t - \theta_p \\
&\quad + \sum_{q_i^{out} \leq q_i^{in}} \delta_{q_i^{in} \to q_i^{out}} r_{\min\{\neg\hat{q}_i^{in\ddagger}, q_i^{out}\}} \\
\mathbb{E}[u_{\rho_i} \mid \hat{q}_i^{in} = 0] &= -\theta_t - \theta_p
\end{aligned}
$$

$\hat{q}_i^{in*} = \hat{q}_i^{in\ddagger}$ when

$$
\begin{aligned}
\mathbb{E}[u_{\rho_i} \mid \hat{q}_i^{in} = \hat{q}_i^{in\ddagger}] &> \mathbb{E}[u_{\rho_i} \mid \neg\hat{q}_i^{in\ddagger} \in (0, \hat{q}_i^{in\ddagger})] \\
n^{in}_{\hat{q}_{i-1}^{out\ddagger} - \neg\hat{q}_i^{in\ddagger}} + \sum_{q_i^{out} \leq q_i^{in}} \delta_{q_i^{in} \to q_i^{out}} [r_{\hat{q}_{i+1}^{in\ddagger}} - r_{\min\{\neg\hat{q}_i^{in\ddagger}, q_i^{out}\}}] &> p_{\hat{q}_i^{in\ddagger}} - p_{\neg\hat{q}_i^{in\ddagger}}
\end{aligned}
$$

and

$$
\begin{aligned}
\mathbb{E}[u_{\rho_i} \mid \hat{q}_i^{in} = \hat{q}_i^{in\ddagger}] &> \mathbb{E}[u_{\rho_i} \mid \hat{q}_i^{in} = 0] \\
v_{q_i^{in}} - p_{\hat{q}_i^{in\ddagger}} + \sum_{q_i^{out} \leq q_i^{in}} \delta_{q_i^{in} \to q_i^{out}} r_{\hat{q}_{i+1}^{in\ddagger}} &> 0.
\end{aligned}
$$

Appendix C: Proof of Proposition 3

Given $q_i^{in} = 0$, the expected utilities of ρ_i given $\hat{q}_i^{in}$ are

$$
\begin{aligned}
\mathbb{E}[u_{\rho_i} \mid q_i^{in} = 0, \hat{q}_i^{in} = \hat{q}_i^{in\ddagger}] &= -\theta_t - \theta_p \\
\mathbb{E}[u_{\rho_i} \mid q_i^{in} = 0, \hat{q}_i^{in} > \hat{q}_i^{in\ddagger}] &= -p_{>\hat{q}_i^{in\ddagger}} - n^{in}_{\hat{q}_{i-1}^{out} - >\hat{q}_i^{in\ddagger}} - \theta_t - \theta_p \\
&\quad + \sum_{\hat{q}_{i+1}^{in} \leq \hat{q}_i^{out*}} Pr(\hat{q}_{i+1}^{in})[r_{\hat{q}_{i+1}^{in}} - n^{out}_{\hat{q}_i^{out*} - \hat{q}_{i+1}^{in}}].
\end{aligned}
$$

$\hat{q}_i^{in*} = \hat{q}_i^{in\ddagger}$ when

$$
\begin{aligned}
\mathbb{E}[u_{\rho_i} \mid q_i^{in} = 0, \hat{q}_i^{in} = \hat{q}_i^{in\ddagger}] &> \mathbb{E}[u_{\rho_i} \mid q_i^{in} = 0, \hat{q}_i^{in} > \hat{q}_i^{in\ddagger}] \qquad (10) \\
0 &> -p_{>\hat{q}_i^{in\ddagger}} - n^{in}_{\hat{q}_{i-1}^{out} - >\hat{q}_i^{in\ddagger}} \\
&\quad + \sum_{\hat{q}_{i+1}^{in} \leq \hat{q}_i^{out*}} Pr(\hat{q}_{i+1}^{in})[r_{\hat{q}_{i+1}^{in}} - n^{out}_{\hat{q}_i^{out*} - \hat{q}_{i+1}^{in}}].
\end{aligned}
$$

By definition, $r_{\hat{q}_{i+1}^{in}} < p_{\hat{q}_i^{in}}$ for all $\hat{q}_{i+1}^{in} \leq \hat{q}_i^{in}$. Thus, the right-hand side of Inequality 10 is negative, and $\hat{q}_i^{in*} = \hat{q}_i^{in\ddagger}$ given $q_i^{in} = 0$.

Bibliography

George A. Akerlof. The market for 'lemons': Quality uncertainty and the market mechanism. *Quarterly Journal of Economics*, 84:488–500, 1970.

Russell Belk. Sharing. *Journal of Consumer Research*, 36:715–734, 2010.

Yochai Benkler. Sharing nicely: On shareable goods and the emergence of sharing as a modality of economic production. *Yale Law Journal*, 114:273–358, 2004.

Rachel Botsman and Roo Rogers. *What's Mine Is Yours: The Rise of Collaborative Consumption*. Harper Business, 2010.

Chrysanthos Dellarocas. The digitization of word-of-mouth: Promise and challenges of online reputation mechanisms. *Management Science*, 49:1407–1424, 2003.

Jeffrey F. Durgee and Gina Colarelli O'Connor. An exploration into renting as consumption behavior. *Psychology and Marketing*, 12:89–104, 1995.

Michal Feldman, Christos Papadimitriou, John Chuang, and Ion Stoica. Free-riding and white-washing in peer-to-peer systems. *IEEE Journal on Selected Areas in Communications*, 24:1010–1019, 2006.

Eric Friedman, Paul Resnick, and Rahul Sami. Manipulation-resistant reputation systems. In Noam Nisan, Tim Roughgarden, Éva Tardos, and Vijay V. Vazirani, editors, *Algorithmic Game Theory*, pages 677–697. Cambridge University Press, 2007.

Garrett Hardin. Tragedy of the commons. *Science*, 162:1243–1248, 1968.

Audun Josang, Roslan Ismail, and Colin A. Boyd. A survey of trust and reputation systems for online service provision. *Decision Support Systems*, 43: 618–644, 2007.

Cait Poynor Lamberton and Randall L. Rose. When is ours better than mine? a framework for understanding and altering participation in commercial sharing systems. *Journal of Marketing*, 2012.

Stephanie J. Lawson. *Forsaking Ownership: Three Essays on Non-Ownership Consumption and Alternative Forms of Exchange*. PhD thesis, Florida State University, 2011.